Math 60 8.6 Compound Inequalities (2 days)

Objectives
1) Review solving linear inequalities in one variable
2) Find the intersection of two sets
3) Find the union of two sets
4) Use set notation for intersection and union correctly
5) Use "and" and "or" correctly
6) Use interval notation for intersections and unions.

Math 60 8.6 Compound Inequalities

① Solve $3x + 2 > -7$. Write solution as interval.

> Isolate variable, the same as for equations
> EXCEPT: if you multiply both sides or divide both sides by a negative number, change the direction of the inequality.

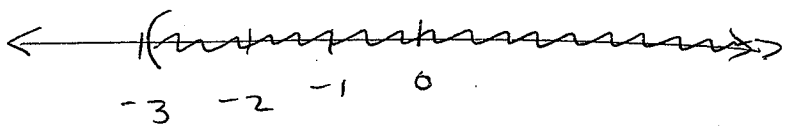
$$3x + 2 > -7$$
$$-2 \quad -2$$

subtract, no change

$$\frac{3x}{3} > \frac{-9}{3}$$

div by positive no change

$$\boxed{x > -3}$$



$$\boxed{(-3, \infty)}$$

② Solve $-2x - 3 \geq 7$

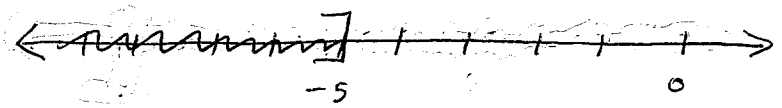
$$+3 \quad +3$$

add, no change

$$\frac{-2x}{-2} \geq \frac{10}{-2}$$

divide by negative * change *

$$\boxed{x \leq -5}$$



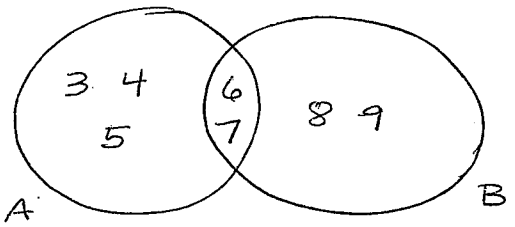
$$\boxed{(-\infty, -5]}$$

Math 60 8.6 Compound Inequalities

(3) Let $A = \{3, 4, 5, 6, 7\}$
and $B = \{6, 7, 8, 9\}$

a) find $A \cap B$

$\cap$ refers to <u>intersection</u>, or overlap

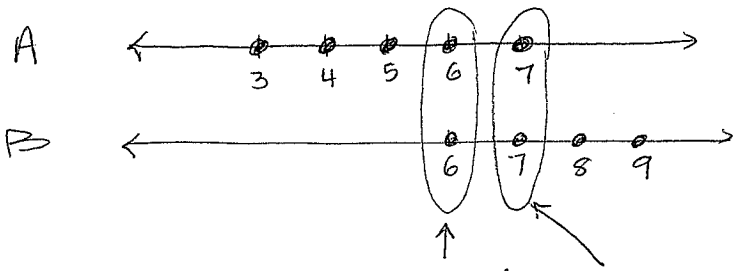


Venn Diagram

set notation always uses curly braces

$\{6, 7\}$ ← 6 is in set A <u>and</u> 6 is in set B
ditto 7.

Another way to see this is using number lines.



6 is in A
<u>and</u> 6 is in B
So 6 is in $A \cap B$

7 is in A
<u>and</u> 7 is in B
So 7 is in $A \cap B$

Note: These individual values are graphed as dots, not shaded regions.
We cannot use interval notation.

$A \cap B = \{6, 7\}$

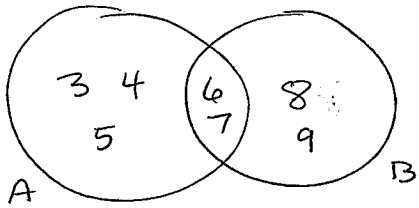
Note: Do not duplicate (repeat) numbers in a set.
List each number only once

Math 60 8.6 Compound Inequalities

b) find $A \cup B$

$\cup$ refers to union, or both sets joined or united.

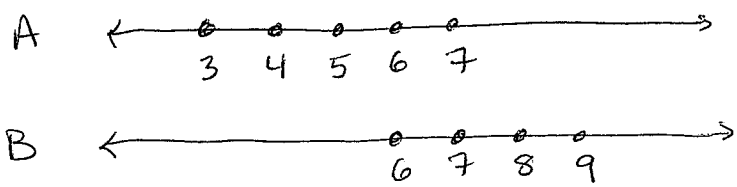
Note: This is not a letter U.



Venn Diagram

$\{3, 4, 5, 6, 7, 8, 9\}$ ← Each of these #s either is in A or is in B (or is in both.)

Another way to see this is using number lines:



This is a logic statement

[the number is in A] or [the number is in B]

"OR" means that if the first statement is true, that's enough.

OR if the second statement is true, that's enough

OR if both statements are true, that's enough.

[3 is in A] → so 3 is in AUB.
true

[6 is in A] OR [6 is in B] → 6 is in AUB
true true

[8 is in B] → 8 is in AUB.

Any # that is ever in either set will be in the union.

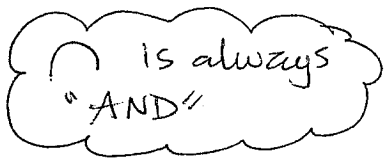
$A \cup B = \{3, 4, 5, 6, 7, 8, 9\}$

Math 60 8.6 Compound Inequality

In summary

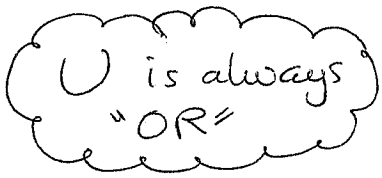
$A \cap B$ = set containing parts that are in A AND are in B.

("shaded twice")
("both shaded")



$A \cup B$ = set containing parts that are in A OR are in B

("shaded once is enough")
("both shaded or one shaded")
("anything that's ever shaded")



Math 60 8.6 Compound Inequalities

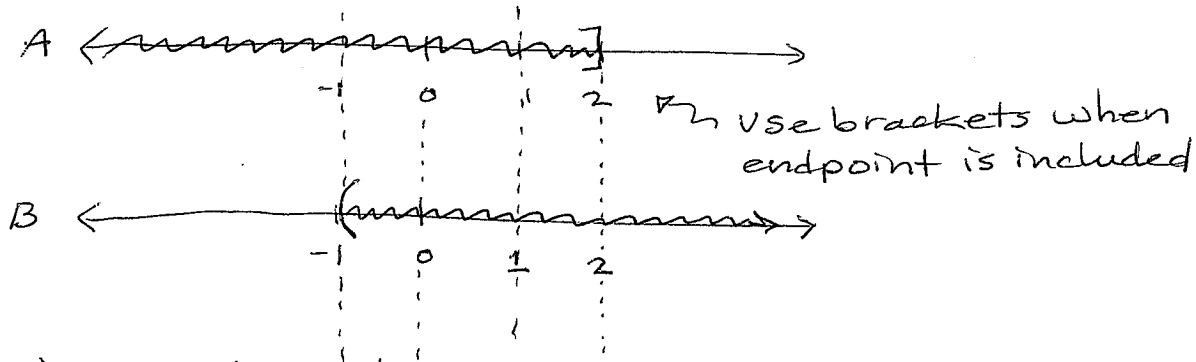
④ Let $A = \{x \mid x \leq 2\}$

$B = \{x \mid x > -1\}$

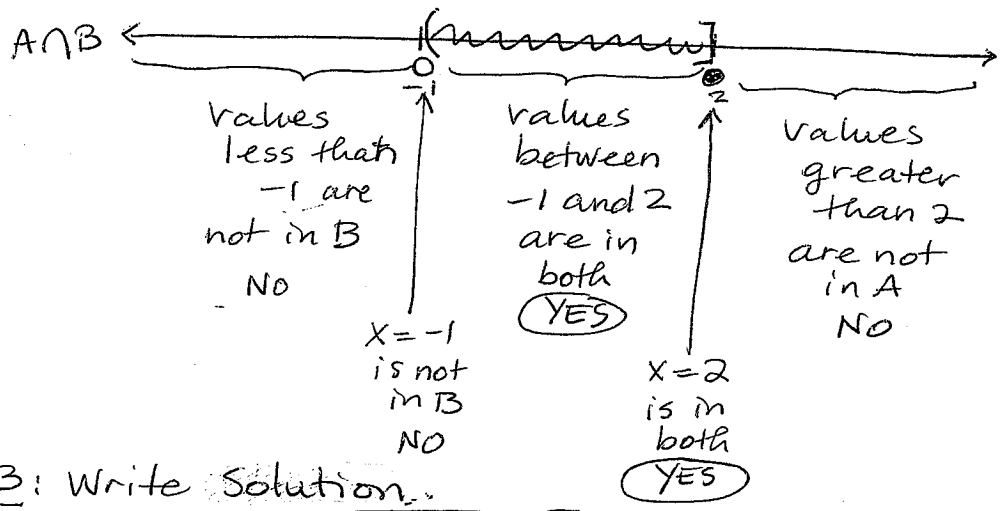
$C = \{x \mid x < -3\}$

a) Find $A \cap B$.

Step 1: Draw a number line, shaded, for each set.



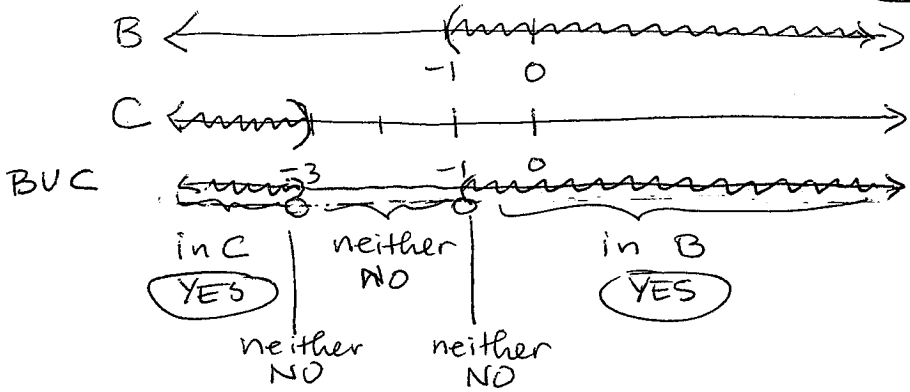
Step 2: Line up all numbers, to determine $A \cap B$ = values which are in A and in B, the overlap.



Step 3: Write solution.

interval notation $A \cap B = (-1, 2]$ $= \{x \mid -1 < x \leq 2\}$ set notation

b) Find $B \cup C = (-\infty, -3) \cup (-1, \infty) = \{x \mid x < -3 \text{ or } x > -1\}$



Math 60 8.6 Compound Inequalities

Let $A = (-\infty, 2]$

$B = (-1, \infty)$

$C = [-3, 4)$

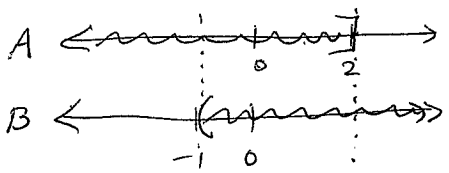
$D = (-5, 3]$

$E = (-\infty, \infty)$

$F = \emptyset$

Write intervals and sets.

Yes ✓ (5) A and B = $A \cap B$

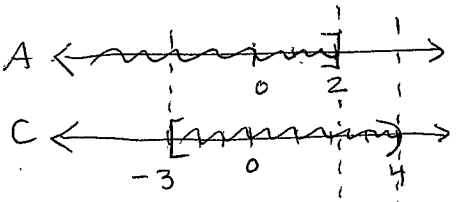


$(-1, 2] = \{x \mid -1 < x \leq 2\}$

Yes Then A OR B

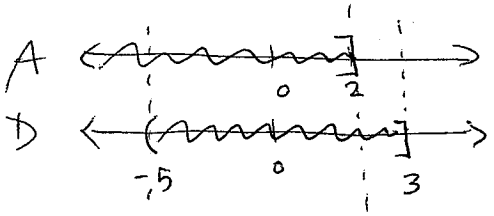
✓ (6) A and C = $A \cap C$

Yes



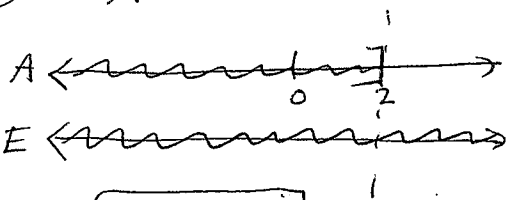
$[-3, 2] = \{x \mid -3 \leq x \leq 2\}$

X (7) A and D = $A \cap D$



$(-5, 2] = \{x \mid -5 < x \leq 2\}$

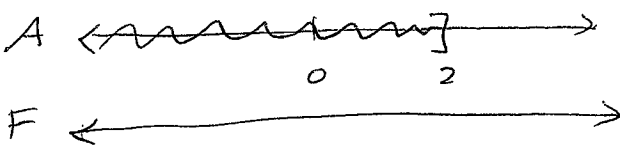
Yes ✓ (8) A and E = $A \cap E$



$(-\infty, 2]$

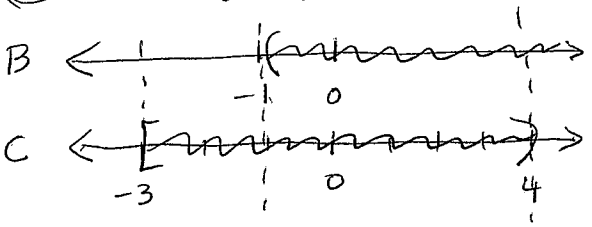
$= \{x \mid x \leq 2\}$

Yes (9) A and F = $A \cap F$



$\emptyset = \{\}$

X (10) B and C = $B \cap C$

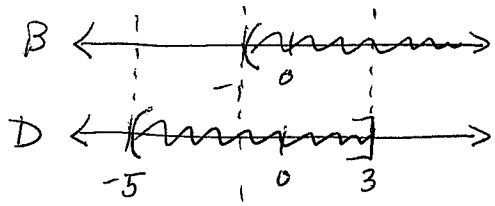


$(-1, 4)$

$= \{x \mid -1 < x < 4\}$

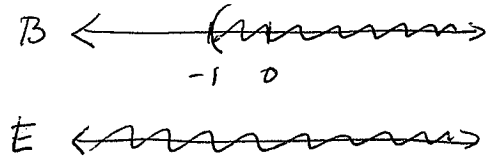
Math 60 8.6 Compound Inequalities

X (11) B and D = $B \cap D$



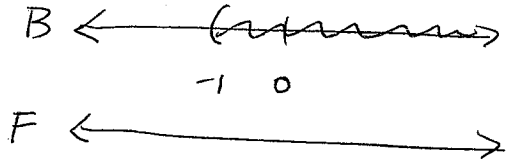
$(-1, 3] = \{x \mid -1 < x \leq 3\}$

✓ (12) B and E = $B \cap E$



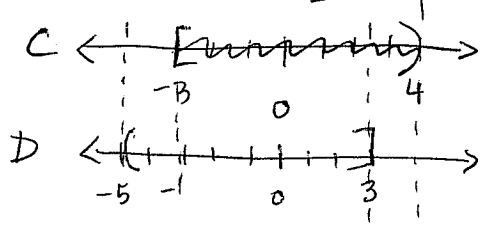
$(-1, \infty)$ $\{x \mid x > -1\}$

X (13) B and F = $B \cap F$



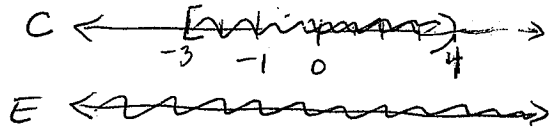
$\emptyset = \{\}$

X (14) C and D = $C \cap D$



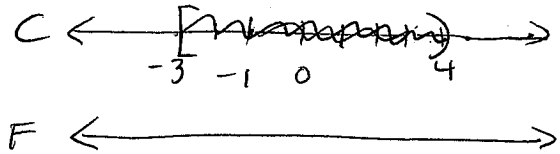
$[-1, 3] = \{x \mid -1 \leq x \leq 3\}$

✓ (15) C and E = $C \cap E$



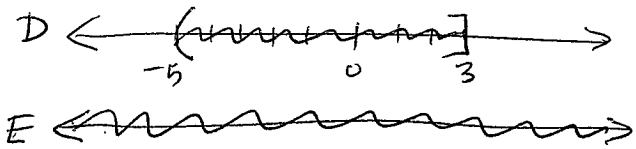
$[-3, 4) = \{x \mid -3 \leq x < 4\}$

X (16) C and F = $C \cap F$



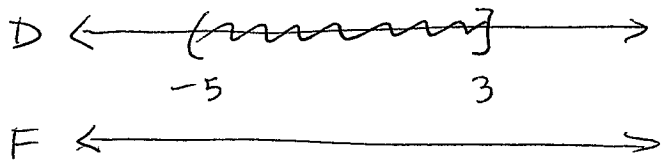
$\emptyset = \{\}$

X (17) D and E = $D \cap E$



$(-5, 3] = \{x \mid -5 < x \leq 3\}$

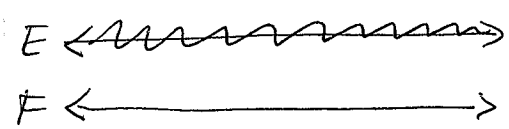
✓ (18) D and F = $D \cap F$



$\emptyset = \{\}$

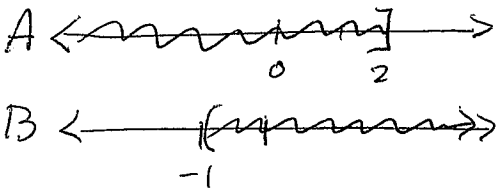
Math 60 8.6 Compound Inequalities

X (19) E and F = E ∩ F



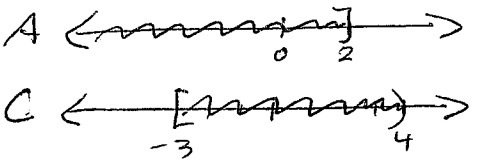
$\boxed{\emptyset} = \boxed{\{\ \}}$

✓ (20) A or B = A ∪ B



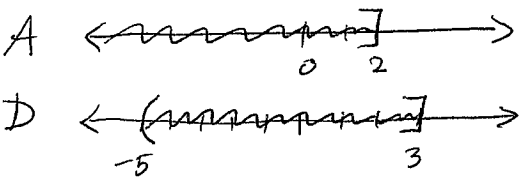
$\boxed{(-\infty, \infty)} = \boxed{\{x \mid x \in \mathbb{R}\}}$

✓ (21) A or C = A ∪ C



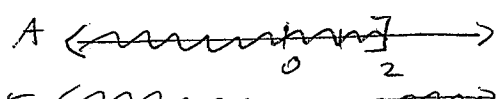
$\boxed{(-\infty, 4)} = \boxed{\{x \mid x < 4\}}$

✓ (22) A or D = A ∪ D



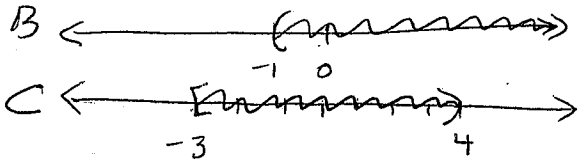
$\boxed{(-\infty, 3]} = \boxed{\{x \mid x \leq 3\}}$

✓ (23) A or E = A ∪ E



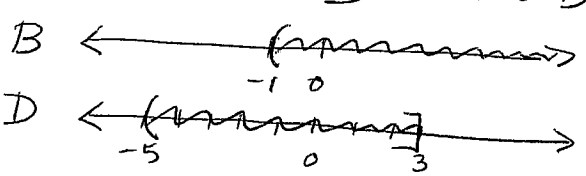
$\boxed{(-\infty, \infty)} = \boxed{\{x \mid x \in \mathbb{R}\}}$

✓ (24) B or C = B ∪ C



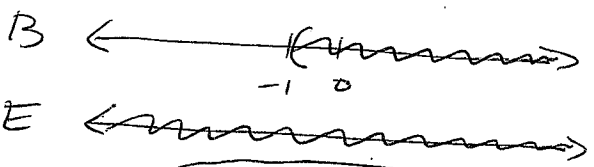
$\boxed{[-3, \infty)} = \boxed{\{x \mid x \geq -3\}}$

✓ (25) B or D = B ∪ D



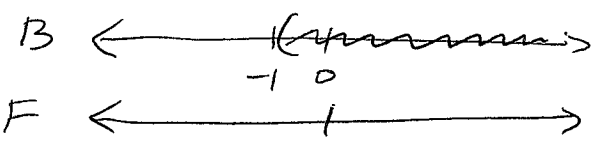
$\boxed{(-5, \infty)} = \boxed{\{x \mid x > -5\}}$

X yes (26) B or E = B ∪ E



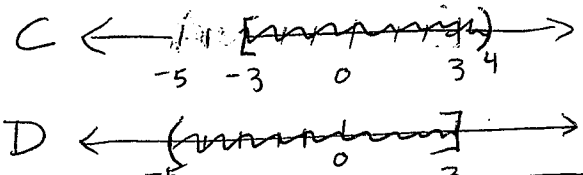
$\boxed{(-\infty, \infty)} = \boxed{\{x \mid x \in \mathbb{R}\}}$

yes ✓ (27) B or F = B ∪ F



$\boxed{(-1, \infty)} = \boxed{\{x \mid x > -1\}}$

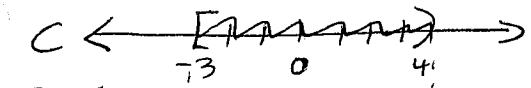
✓ (28) C or D = C ∪ D



$\boxed{(-5, 4)} = \boxed{\{x \mid -5 < x < 4\}}$

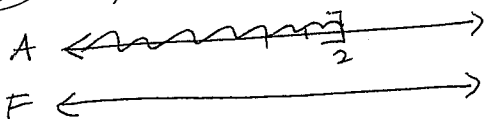
Math 60 8.6 Compound Inequalities

X (29) C or E = C ∪ E



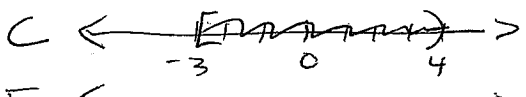
$(-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$

X (34) A or F = A ∪ F



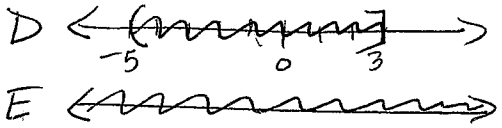
$(-\infty, 2] = \{x \mid x \leq 2\}$

X (30) C or F = C ∪ F



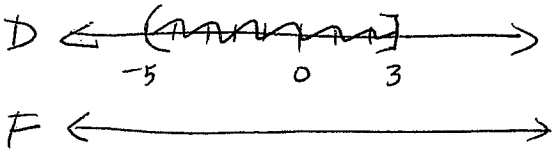
$[-3, 4) = \{x \mid -3 \leq x < 4\}$

X (31) D or E = D ∪ E



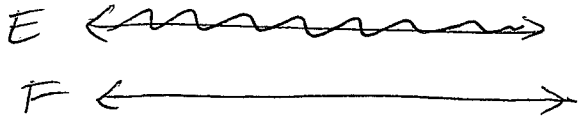
$(-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$

X (32) D or F = D ∪ F



$(-5, 3] = \{x \mid -5 < x \leq 3\}$

✓ (33) E or F = E ∪ F



$(-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$